

Joint Estimation of Information and Distributed Link-Scheduling in Wireless Networks: Mean-Field Approximation and Graphical Models

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Abstract—In a large multi-hop wireless network, nodes are preferable to make distributed link-scheduling decisions with information exchange only among a small number of neighbors. However, for a slowly-decaying channel and densely-populated interfering nodes, a small size neighborhood often results in nontrivial link outages and is thus insufficient for making optimal scheduling decisions. A question arises how to incorporate the information outside a neighborhood in distributed link-scheduling. In this work, we develop joint approximation of information and distributed link scheduling. We first apply machine learning approaches to model distributed link-scheduling with complete information. We then characterize the information outside a neighborhood, i.e., the residual interference, as an aggregated random loss variable. The loss variable is characterized by either a Mean-Field approximation or a normal distribution based on the Lyapunov central limit theorem. The approximated information outside a neighborhood is incorporated in a factor graph. This results in joint approximation of information and distributed link-scheduling in an iterative fashion. Link-scheduling decisions are first made at each individual node based on the approximated loss variables. Loss variables are then updated and used for next link-scheduling decisions. The algorithm repeats between these two phases until convergence. Interactive iterations among these variables are implemented by a message-passing algorithm over a factor graph. Simulation results show that using learned information outside a neighborhood jointly with distributed link-scheduling reduces the outage probability close to zero even for a small neighborhood.

Index Terms—Distributed Link-Scheduling, Wireless Networks, Mean-field approximation, Lyapunov central limit theorem, Message Passing, Factor Graph.

I. INTRODUCTION

Efficient link-scheduling and channel assignment are fundamental to optimal resource utilization in wireless networks [5][14][22]. Distributed scheduling is a frequent scenario, where individual nodes make scheduling decisions using only information from their neighbors [2][21][22][33].

Such distributed scheduling is optimal if an objective function to be optimized is local by nature, i.e., decomposable to factors with nodal variables and their neighbors [21][22][33]. Distributed scheduling with local information-exchange becomes non-optimal if information outside neighborhoods is pertinent. For example, interference causes long-range spatial dependence in multi-hop wireless networks [16][28]; thus optimal scheduling decisions made by nodes require information not only within neighborhoods but also outside.

In our prior work, we studied such a scenario where there was a long range spatial dependence among nodes but the

dependence was weak. We developed distributed scheduling based on a machine learning framework, where information exchange occurred only among neighbors. Information, i.e., on residual interference, was entirely ignored outside a neighborhood range. We derived a sufficient condition to show that the aggregated spatial dependence outside a neighborhood needed to be weak, i.e., decay sufficiently fast, for such a distributed algorithm to be near-optimal.

This work studies the scenarios that the spatial dependence is of long-range and strong. The information, if ignored outside neighborhoods, would result in a significant link outage probability. Therefore, distributed scheduling with information exchange only from neighbors is no longer near-optimal[16]. This corresponds to the scenario of either a slowly-decaying channel or densely populated interferers. There, the aggregated residual interference cannot be ignored [1][3][7][28][16]. Hence, a node needs to possess information outside its neighborhood, e.g., on residual interference, when making cooperative scheduling-decisions. Otherwise, decisions on channel-access made by a node may cause significant interference even at faraway nodes in a network. In our prior work [16], we show that residual interference outside a neighborhood cannot be ignored especially for slowly decaying channels and densely populated nodes. However, how to incorporate the information outside a neighborhood is not studied there. Overall, systematic study is also lacking on what size neighborhood is sufficient and what information to include for optimal distributed link-scheduling.

One scenario is for nodes to simply include complete information outside their neighborhood. For example, a protocol model based on contention [12] assumes that each node maintains the configuration information from an entire network. This, however, does not reflect a realistic communication environment [16], where complete information is often unavailable at individual nodes. Even if the complete information were available, a node would need to exchange information with all others [7][9][27], which can be prohibitive in a large network. As an alternative, we consider aggregated information outside the neighborhoods as an approximation.

The approximation does not consider information from individual nodes outside a neighborhood but their aggregated contribution. Such approximation needs to satisfy a certain optimality in regard to an underlying exact model. Hence, we first provide a network model for distributed link-scheduling. The model is developed in our prior work [16], and based

on Boltzmann distribution and probabilistic graphical models (see [10] [17] and reference therein). The model includes long-range spatial dependence among all nodes in a network, and is exact under given assumptions.

We then consider two approximations to the long-range dependence, i.e., residual interference, outside a neighborhood range. Both approximations regard the information outside a neighborhood as one aggregated variable. In particular, the first approximation views the aggregated residual interference as a deterministic mean-field. The mean-field parameters optimize a variational bound of the probabilistic graph of our network model [20]. The parameter values are obtained through solving coupled mean-field equations [17][20]. Mean-field approximations have been used widely in machine learning and image processing [17] in the context of graphical models. In networking, Mean-Field approximation has been applied to loss networks [18]. These forms of Mean-Field approximations are derived based on an underlying model of networks, which motivate our work here. For wireless network, Mean-Field has been considered as a limiting form of a large number of mobile users [4]. This is related to the idea of aggregating dependencies outside a neighborhood in this work. Mean-Field theory is also used to derive effective link capacity of wireless networks [19].

The second approximation treats the aggregated residual interference as a random variable, referred to as loss variable. We show that the loss variable can be approximated by a normal distribution through the Lyapunov central limit theorem (CLT) [32]. The mean and the variance of the normal distribution can be learned using information within a neighborhood range. The approximations are combined with iterative and distributed link-scheduling decisions. In particular, a message-passing algorithm on the factor graph [23] is combined with the approximations of the long-range residual interference for the distributed and statistical link-scheduling. The mean-field parameters and the normal residual random variable form particular local functions in the factor graph. The Mean Field approximation or the distribution of the loss variables are learned iteratively from the available information within the neighborhood range. Distributed decisions are then made through alternating between approximation and message passing over the factor graph. This process repeats until convergence.

We then conduct simulation on network-wide link outage probability to complement the analytical derivation of the approximation and the algorithm. Simulation results show that distributed link-scheduling with an arbitrarily small neighborhood and aggregated information outside can satisfy the global SINR requirements while maximizing the channel reuse.

The rest of the paper is composed as follows. Section II provides motivating examples on distributed scheduling using complete, partial and approximated information. In Section III, we provide a problem formulation. Section IV provides an accurate probabilistic model of distributed link-scheduling decisions at individual nodes. In Section V, we obtain two approximations on residual interference, and the resulting simplified probabilistic graphical models. In Section VI, we derive distributed message-passing algorithm coupled with the

approximation. In Section VII, we provide simulations to show the performance of the algorithms using two approximations. Section VIII concludes this study and discusses open questions.

II. DISTRIBUTED LINK-SCHEDULING WITH LOSS VARIABLES

In this section, we provide motivation on the importance of information outside a neighborhood. We first describe the setting of distributed link-scheduling. We then present an example of distributed scheduling with complete information. We provide an example how the performance degrades for distributed scheduling with partial information. This motivates two approximation schemes. We provide an overview on the approximation schemes to motivate the subsequent sections.

A. Distributed Link-Scheduling Decisions

In a distributed setting, a node determines its channel access schedule, i.e., whether to access a channel at a time slot, using the information within its neighborhood. The information is on the “state that includes the relative positions, the transmission statues (who is transmitting to whom), transmission power, and scheduling decisions of links. For simplicity, this work does not consider power control and assumes fixed transmission power for each node.

The decision made is cooperative and based on two criteria. The first is whether the SINR requirements are satisfied at the neighboring links if a node chooses to use a channel. The second is to maximize the spatial channel-reuse which is the number of concurrently active links in the network.

Decisions are made in a distributed fashion. A node sends its decision to neighbors. The neighbors use the information to make their own link-scheduling decisions, and then send the decisions to their neighbors. Distributed decisions are made iteratively and collectively by all nodes in a network until convergence.

An assumption is that a node knows complete information from its neighbors. Hence, the neighborhood size relative to the entire network determines whether the information used by a node is complete or partial.

B. Complete Information

First, we consider an ideal setting, where complete information is available for individual nodes to make link-scheduling decisions [8][30]. The complete information is on the state from all nodes in a network. That is, the neighborhood for any node is an entire network. A node then determines whether to access a channel at a timeslot using the available information. Such a scenario is used as a baseline for comparing approximations.

Consider an example of a linear network in Figure 1. Assume that the channel-contention constraint requires any active link between two adjacent nodes to be separated by one neighboring link. The interference constraint requires any active link to be separated by at least two links. Assume

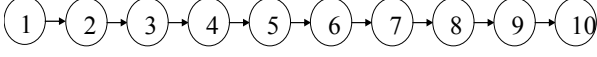


Fig. 1. A Linear Topology with 10 Nodes and 9 Directional Links.

also that the network achieves the spatial channel-reuse maximization where the total number of concurrent active links is maximized.

Figure 2 shows all possible configurations of active links. Each row corresponds to a snapshot of active links at a time epoch, where the active links satisfy the aforementioned contention and interference constraints. For example, the first row denotes a configuration where both link (3,4) and (8,9) are active. Overall, there are 2 configurations with two active links. The remaining 8 configurations have three active links that result in the maximum channel reuse. Note that for distributed decisions where each node has complete information on the activities of the others, the configurations with three active links are ones that maximize the channel reuse while satisfying the constraints.

C. Partial Information

Partial (state) information is available to a node when its neighborhood consists of only nodes from parts of a network.

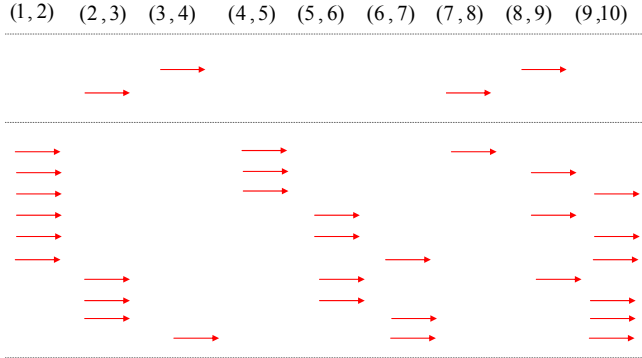


Fig. 2. Possible configurations. “→”: Active link

For example, assume the neighborhood for each node is the adjacent link. This means that the neighborhood is only determined by the contention constraint where a node would not access the channel if its neighbor is transmitting. Such a neighborhood size is smaller than desirable since node does not know whether SINR is satisfied at its neighbors’ neighbors. Hence, the SINR constraint may be violated.

Specifically, under the above assumption on the neighborhood size, it is legitimate for node 3 to transmit to node 4 when links (1,2), (6,7) and (9,10) are active (see the eighth row in Figure 2). Those active links are outside the neighborhood system and thus invisible to node 2. Active link (3,4) thus generates interference, violating the SINR constraints at the existing active links.

By increasing the neighborhood size, we show, in our prior work [16], that an optimal configuration with three active links can be obtained through fully distributed node decisions with

a chosen neighborhood size. For example, if two links (2,3) and (7,8) are active at the beginning, a configuration with 3 active links can be obtained if the distributed algorithms are randomized, i.e., nodes make probabilistic decisions for transmission. Our prior work also derives sufficient conditions for such distributed scheduling to be near-optimal. However, our prior work does not study sufficiently when partial information in distributed scheduling is insufficient for an obtaining optimal configuration.

D. Importance of Information Outside Neighborhood

The above example shows that for a finite and often small number of neighbors, the SINR requirement is not guaranteed [16][28]. In other words, information within a neighborhood can be insufficient for making correct distributed decisions when the information outside the neighborhood is neglected.

To further illustrate the importance of information outside a neighborhood, we consider another example of a multi-hop wireless network. The network has nodes located in a square area of 10 x 10 square meters, $L = 200$ links, $\alpha = 4$ as the channel attenuation factor, and SINR threshold $\text{SINR}_{th} = 10$. Here $\alpha = 3$ corresponds to slow power decay of the channel, resulting in strong interference and thus dependence among nodes. Such a scenario is not studied in our prior work [16], where the strong dependence violates the sufficient condition we derive for distributed scheduling with a moderate neighborhood to be near-optimal.

Suppose we ignore the residual interference outside a neighborhood range, which is common practice for large wireless networks. We then calculate the link outage probability that the SINR requirement is violated at a link. The outage probability is shown to be significant and not negligible as in Figure 3, especially for a moderate neighborhood size.

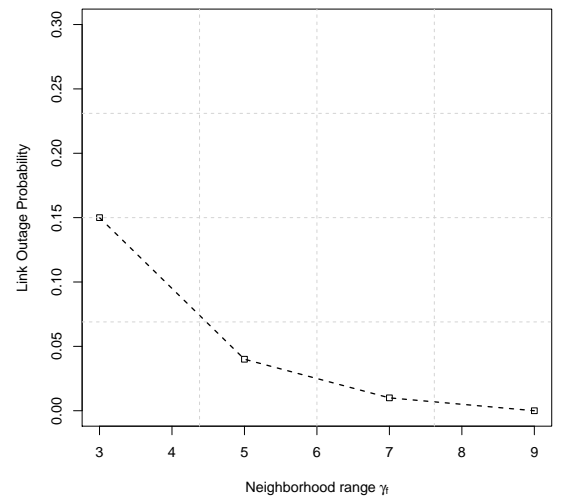


Fig. 3. Link Outage probability.

Note that the link outage due to the ignored residual interference is different from the transient outage due to the fading channel. The link outage here is persistent. Thus, the

network connectivity may not be guaranteed. Some nodes can even be permanently disconnected if outages occur persistently at adjacent links.

This example clearly shows that the residual interference cannot be arbitrarily ignored in a large wireless network.

E. Approximation of Information Outside Neighborhood

Now consider that each node accounts for information outside its neighborhood, i.e., the residual interference, as a loss variable. The loss variable treats the impact from the rest of a network except the neighbors in aggregation.

Two approaches are used for the approximation. The first is to characterize the expected value of a loss variable through the deterministic Mean-Field approximation [24]. That is, a set of Mean-Field equations are obtained through optimizing a variational bound from the network model [20]. The expected values of the loss variables can be obtained iteratively through solving deterministic Mean-Field equations. The second is to regard the loss variable as random and estimate its probability distribution. The probability distribution can be learned iteratively using information within the neighborhood.

The approximations are incorporated into distributed scheduling decisions. Each node first estimates either the Mean-Field value or the distribution of the loss variable. The node then determines channel-access schedule using both complete information within its neighborhood and estimated information on the loss variable. The algorithm is to be described in Section VI.

III. PROBLEM FORMULATION

We now obtain a problem formulation for distributed scheduling with loss variables.

A. Notations

Consider a wireless multi-hop network with n nodes. X_i and P_i denote the position and the transmission power of node i , respectively. Let $\mathbf{X} = \{X_1, \dots, X_n\}$, where bold characters are used to denote vectors in the rest of the paper. Let \mathbf{L} denote a set of communication links that access a common wireless channel. Let σ_{ij} represent the activity of link $(i, j) \in \mathbf{L}$ at a time instance: $\sigma_{ij}=1$ if link (i, j) is active, i.e., using the common wireless channel; $\sigma_{ij}=0$, otherwise. Let $\boldsymbol{\sigma} = \{\sigma_{ij}\}$ where $(i, j) \in \mathbf{L}$. Here node i is assumed to be a transmitter whereas node j is a receiver.

We assume that each node maintains complete information within a neighborhood range. The shape of the neighborhood for each node can be arbitrary rather than circular. The neighborhood range can be different for all nodes but bounded by a radius γ_f . Hence, without loss of generality, γ_f is assumed to be the neighborhood range for each node.

The channel gain for link (i, j) between node i and j is denoted as $G(i, j)$. For simplicity, $G(i, j) = |X_i - X_j|^{-\alpha}$, where α is the channel power attenuation factor and $2 \leq \alpha \leq 6$. $G(i, j)$ is random when positions X_i 's are assumed random.

R_{ij} is the inverse of SINR of an active link (i, j) at a time instance,

$$R_{ij} = \frac{\sum_{mn \neq ij} P_m G(m, j) \sigma_{mn} + \eta_j}{P_i G(i, j)}, \quad (1)$$

where η_j denotes the noise power at the receiver j . For simplicity, all nodes assume to have the same (random) noise power $\eta_j = N_b$. To satisfy the SINR constraint, $R_{ij} \leq R_{th}$ holds, where $R_{th} = 1/\text{SINR}_{th}$ is the inverse of a required SINR threshold SINR_{th} . Let R_{ij}^l be the approximation of R_{ij} that includes complete information within the neighborhood of node i plus the residual interference outside,

$$R_{ij}^l = \frac{\sum_{|X_m - X_j| \leq \gamma_f} P_m G(m, j) \sigma_{mn} + \eta_j + \text{Res}_{ij}}{P_i G(i, j)}, \quad (2)$$

where Res_{ij} denotes the approximated value of residual interference outside the neighborhood range experienced by the receiver j of active link (i, j) . The distance between the closest interferer and the receiver of the active link is referred to as the contention range r_c of an active link.

B. Objectives

The objective of link scheduling is to maximize the spatial channel-reuse at a time instance, while satisfying the SINR requirement of active links, i.e.,

$$\begin{aligned} & \text{maximize} && \sum_{ij} (R_0 - R_{ij}) \sigma_{ij} \\ & \text{subject to} && R_{th} \geq R_{ij}, \text{ for } \forall \sigma_{ij} = 1, \end{aligned} \quad (3)$$

where R_{ij} is the inverse of the interference at receiver j from (1). R_{th} is the inverse of an interference threshold. R_0 is a large positive constant where $R_0 > R_{ij}$ for $\forall (i, j) \in \mathbf{L}$. R_0 is used for the objective function to be maximized whenever a new link becomes active. The first term $\sum_{ij} \sigma_{ij}$ is the total number of active links which should be maximized for the channel reuse in the network. The second term $\sum_{ij} R_{ij} \sigma_{ij}$ represents the effect of total interference from active links.

The objective function in (3) satisfies the following three properties. The objective function is (i) constrained to satisfy the SINR requirement of any active links; (ii) monotonically increasing with respect to the number of active links; and (iii) monotonically increasing as the inverse SINR of an active link decreases. Therefore, the two terms in the objective function make trade-offs between the channel reuse and the SINR requirements.

To measure the optimality of the objective function, we use the link outage probability $P(\text{SINR}_{ij} < \text{SINR}_{th})$. As soon to be shown, a limited neighborhood size is a major cause of significant link outage probability considered in this work.

C. Probabilistic Decisions

Optimization is achieved through distributed decisions at individual nodes. Nodes determine whether and when σ_{ij} 's become active. Decisions made by a node need to account for loss variables, i.e., the aggregated information outside its

neighborhood, and decisions made by neighbors. As deterministic distributed decisions can be easily trapped in local minima, we consider probabilistic, i.e., randomized decisions (see [10] [13] [16] and references therein). At each time epoch, nodes make the link-scheduling decisions based on a probabilistic model of the neighboring nodes' link-scheduling decisions at the previous time epoch and the loss variable. This requires a probabilistic decision model.

IV. PROBABILISTIC MODEL

The probabilistic model of distributed link-scheduling decisions is based on a machine learning framework in form of a Boltzmann distribution and a dependency graph. The model is drawn from our prior work [16], and included here for completeness.

A. Boltzmann Distribution

Our model considers σ_{ij} 's as a random field of link activities. Such a random field resembles a particle system with interacting particles (spins) [34]. States of particles can be viewed as binary random variables. A configuration ω of interacting binary-state particles can be described by a system potential energy $H(\omega)$. The probability distribution of $H(\omega)$ obeys the Boltzmann distribution $P(\omega) = Z_0^{-1} \cdot \exp(-H(\omega))$ [10][34], where $Z_0 = \sum_{\omega} \exp(-H(\omega))$ is a normalizing constant.

We now regard the objective (3) as the system potential energy and appeal to Boltzmann law in statistical physics [10][16][25][33][34]. In particular, the system potential energy of binary link-scheduling decisions is defined as,

$$\begin{aligned} H(\sigma) &= \sum_{ij} (-R_0 + R_{ij})\sigma_{ij} + \beta \sum_{ij} U(R_{ij} - R_{th})\sigma_{ij} \quad (4) \\ &= \sum_{ij} (-R_0 + \frac{N_b}{P_i l_{ij}^{-\alpha}})\sigma_{ij} \\ &\quad + \sum_{ij} \sum_{mn \in N_{ij}} \frac{P_m l_{mj}^{-\alpha}}{P_i l_{ij}^{-\alpha}} \sigma_{ij} \sigma_{mn} \\ &\quad + \sum_{ij} \frac{h_{ij}}{P_i l_{ij}^{-\alpha}} \sigma_{ij} + \beta \sum_{ij} U(R_{ij} - R_{th})\sigma_{ij}, \end{aligned}$$

where $\beta > 0$ is a constant. $\beta U(R_{ij} - R_{th})$ is a penalty term for the SINR constraint. N_{ij} denotes the set of links within the neighborhood range from receiver j of link (i, j) , and h_{ij} denotes total residual interference outside the neighborhood range experienced by active link (i, j) , i.e., $h_{ij} = \sum_{mn \notin N_{ij}} P_m l_{mj}^{-\alpha} \sigma_{mn}$.

From Boltzmann law [10][16], the probabilistic model for a set of link-scheduling decisions at a time instance is,

$$P(\sigma) = Z_{\sigma}^{-1} \cdot \exp(-H(\sigma)), \quad (5)$$

where σ is a random field (i.e., configuration) of all link activities σ_{ij} 's. $Z_{\sigma} = \sum_{\sigma} \exp(-H(\sigma))$ is a normalizing constant. Note that both the number of active links and interference play important roles: If multiple configurations exist with the same number of active links, the system potential energy distinguishes different configurations by the sum of the inverse SINR of active links.

A desirable configuration corresponds to a small system potential energy, and a large Boltzmann probability [10]. Hence, an optimal decision σ^* is the one that maximizes the Boltzmann distribution,

$$\sigma^* = \arg \max_{\sigma} P(\sigma). \quad (6)$$

B. Dependency Graph

The probability model exhibits a graphical representation [16][17][23]. The graphical representation shows the statistical spatial dependence among link activities. For ease of illustration, consider a directional linear topology of eight nodes and seven directional links, where the directions correspond to a data flow. The corresponding dependency graph of link-scheduling decisions σ is shown in Figure 4.

Nodes in the graph represent binary random variable $\{\sigma_{ij}\}$ for $\sigma_{ij} \in \{0, 1\}$ and $1 \leq i \leq 7$ and $j = i + 1$. A link between two nodes represents the spatial dependence. The spatial dependence results from interference in the wireless channel and corresponds to coupled terms in the system potential energy $H(\sigma)$. The dependency graph can be further represented in form of a factor graph shown in Figure 5. A factor graph consists of variable nodes and functional nodes [23]. Here, σ_{ij} denotes a variable node for link $(i, j) \in L$. A functional node $\psi_{ij} = R_{ij}$ denotes a potential function of σ_{ij} .

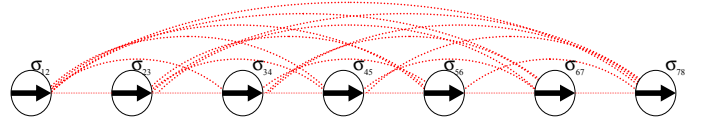


Fig. 4. Dependence Graph of random field σ

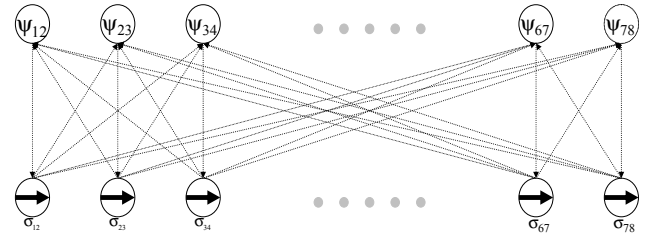


Fig. 5. Example Factor Graph Representation.

Link-scheduling decisions are all dependent due to interference. Thus the dependency graph is fully connected. Such a fully connected graph shows that the neighborhood of a node is in fact the entire network.

C. Optimal Configuration and Distributed Decisions

A fully connected dependency graph impacts how distributed decisions should be made using the probabilistic model. Node decisions are all dependent. Hence, optimal

decisions require nodes to exchange information with all the other nodes in the network. Graphically, this means that an entire network is a neighborhood. That is, a node needs to obtain information on all other nodes. This, in reality, prohibits implementation of optimal distributed link-scheduling.

V. APPROXIMATION OF PROBABILISTIC MODEL

We now derive approximations of the probabilistic model for distributed scheduling. Our approximation considers a moderate neighborhood for each node where the node has complete information on its neighbors. Outside the neighborhood, the residual interference, instead of being ignored, is characterized as a random loss variable. Either the mean values or the distributions of the loss variables are then included in the probabilistic model. The resulting model $P^l(\sigma)$ is an approximation of the exact model $P(\sigma)$.

A. Approximation by Mean Field

We first characterize the residual interference by Mean Field (MF) approximation. The MF approximation optimizes a variational bound between the approximated and the exact models [20], resulting in a set of deterministic mean-field equations. The mean-values of the loss variables can then be obtained through solving the Mean-Field equations iteratively. To derive the Mean-Field equations, we consider the system potential energy in (4). For simplicity, we assume that the SINR constraint is satisfied so that the system potential energy is reduced to

$$\begin{aligned} H(\sigma) &= \sum_{ij} f_{ij}(\sigma) \\ &= \sum_{ij} a_{ij} \sigma_{ij} + \sum_{ij} \sum_{mn \in N_{ij}} a_{ij,mn} \sigma_{ij} \sigma_{mn} \\ &\quad + \sum_{ij} h_{ij}, \end{aligned} \quad (7)$$

where $a_{ij} = -R_0 + \frac{N_b}{P_i l_{ij}^{-\alpha}}$, $a_{ij,mn} = \frac{P_m l_{mj}^{-\alpha}}{P_i l_{ij}^{-\alpha}}$ from (4), and $h_{ij} = \sum_{mn \notin N_{ij}} a_{ij,mn} \sigma_{ij} \sigma_{mn}$ corresponds to the aggregated residual interference outside the neighborhood for link (i, j) . Note that this system potential energy represents a second-order Ising model in statistical physics, where all terms are dependent.

Let h_{ij}^* be a deterministic approximation of h_{ij} . An approximated system potential energy absorbs the impact of the aggregated residual interference into the coefficient a_{ij} 's,

$$\begin{aligned} H^l(\sigma) &= \sum_{ij} g_{ij}(\sigma) \\ &= \sum_{ij} b_{ij} \sigma_{ij} + \sum_{ij} \sum_{mn \in N_{ij}} a_{ij,mn} \sigma_{ij} \sigma_{mn}, \end{aligned} \quad (8)$$

where $b_{ij} = a_{ij} + h_{ij}^*$, and h_{ij}^* is yet to be found.

We denote the true free energy [20] with F ,

$$\beta F = -\ln(Z_P), \quad (9)$$

where $\beta > 0$ is a constant, and $Z_P = \sum_{\sigma} \exp(-H(\sigma))$. Let variational free energy [20] be \tilde{F} ,

$$\beta \tilde{F} = -\ln(Z_Q), \quad (10)$$

where β is a large positive constant, and $Z_Q = \sum_{\sigma} \exp(-H^l(\sigma))$.

To find the simplest, i.e., the first-order Mean Field (MF) approximation, we make the following assumptions: (a) for link $(i, j) \neq (m, n)$, the impacts only affect the first order terms; (b) for $(m, n) \notin N_{ij}$, σ_{ij} is independent of σ_{mn} ; and (c) for the approximated system potential energy, the corresponding distribution is $Q(\sigma) = Z_Q^{-1} \exp(-H^l(\sigma))$. The mutual information measure between the approximated and the true distribution results in an objective function, i.e., a variational free energy [20],

$$\begin{aligned} \beta \tilde{F} &= E_Q(\ln(\frac{Q(\sigma)}{P(\sigma)})) - \ln(Z_P) \\ &= E_Q(\ln(H(\sigma)) - \ln(H^l(\sigma))) - \ln(Z_Q) \\ &= -\ln(Z_Q) + \sum_{ij} (-h_{ij} \mu_{ij}) + \sum_{ij} \sum_{mn \notin N_{ij}} a_{ij,mn} \mu_{ij} \mu_{mn}, \end{aligned} \quad (11)$$

where $\mu_{ij} = E(\sigma_{ij}) = P(\sigma_{ij} = 1)$. Inserting Equation (10) in (11) and letting $\frac{\partial \tilde{F}}{\partial h_{ij}} = 0$, the optimal value of h_{ij} satisfies

$$h_{ij}^* = \sum_{mn \notin N_{ij}} a_{ij,mn} \mu_{mn}. \quad (12)$$

h_{ij}^* is a weighted sum of expected values of link activities outside the neighborhood range. Hence, h_{ij}^* can be regarded as the mean field outside the neighborhood of link (i, j) . As a special case when $a_{ij,mn} = 0$ for $mn \notin N_{ij}$, $h_{ij}^* = 0$.

For $(i, j) \in L$, there are n such equations, where n is the total number of nodes in a network. Those equations are coupled. The coupling can be shown explicitly by rewriting h_{ij}^* as $h_{ij}^* = \log \frac{1 - \mu_{ij} \eta_1}{\mu_{ij} \eta_2}$. Equation (12) then takes the following form,

$$\log \frac{1 - \mu_{ij} \eta_1}{\mu_{ij} \eta_2} = \sum_{mn \notin N_{ij}} a_{ij,mn} \mu_{mn}, \quad (13)$$

where η_1 and η_2 are functions of h_{kl}^* 's for $(k, l) \neq (i, j)$ (and in turn μ_{kl} 's). Hence, the set of n equations are coupled through unknown values of h_{ij}^* 's or μ_{ij} 's. Appendix I provides detailed derivations.

The coupled Mean-Field equations can be solved iteratively and numerically starting from an initial condition [20]. The Mean-Field entity h_{ij}^* can also be estimated when measurements are available. For example, the interference within the neighborhood range can be known with the information exchange. Thus, if we assume that the aggregated interference can be measured possibly with a measurement error, h_{ij}^* can be estimated when interference measurements are available.

An advantage of the Mean-Field approximation is that a resulting probability model is a simple Markov random field, where the deterministic Mean-Field values h_{ij}^* 's absorb the long-range spatial dependence in the residual interference. However, the Mean-Field approximation exhibits two disadvantages. First, it can be computationally intensive to solve a set of Mean-Field equations. Second, if the Mean-Field parameters are estimated by measurements instead, such measurements on the aggregative residual interference are either unavailable or computationally intensive to obtain. For example, such measurements require nodes to transmit test signals during link-scheduling decisions. In addition, nodes need to be capable of measuring the aggregated interference.

For example, packet reception ratio (PRR) versus SINR models need to be seeded by $O(n)$ trials in an n -node network where each node transmits while receivers measure the channel conditions [26].

B. Approximation by Lyapunov Central Limit Theorem

An alternative approach is to treat the aggregated residual interference as random and approximate its distribution. Such an approximation can be computationally less expensive to obtain than the Mean-Field approximation.

Let n_{ij} be a random variable that approximates the residual interference of an active link (i, j) . Our goal is to identify a probability distribution for n_{ij} . As residual interference aggregates interference signals from many interferers, one option is to approximate n_{ij} as a normal random variables.

We first obtain an expression for n_{ij} to understand whether a normal approximation is feasible. Based on our assumptions, interferers have the same transmission power P_0 . Locations of interferers outside a neighborhood range can be arbitrary/random but follow certain patterns of node-positions within the neighborhood. In other words, the network topology exhibits a certain regularity. Specifically, the following assumptions are posed on the location of interferers.

- For an active link (i, j) , all interferers are located uniformly within radial bands bounded by concentric circles from receiver j . The radius of the circles exhibits randomness, i.e., $r_k = r_{k-1} + x_k$ for $k \geq 1$, where $r_0 = \gamma_f$, and $x_k = x$ is a uniform random variable between r^l and r^u . Here, r^l and r^u are the minimum and maximum distance between any two active links, within the neighborhood of link (i, j) . The circle with radius r_k is called the k -th frontier.
- On the k -th frontier for $k \geq 1$, any two neighboring active links are separated by random distance x_0 . Random variables x_k and x_0 are independent and identically distributed.

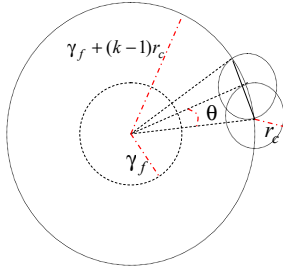


Fig. 6. Configuration of Active Links outside Neighborhood range.

Lemma 1: With the above two assumptions, the residual interference h_{ij} for active link (i, j) can be modeled as

$$n_{ij} = \sum_{k=1}^{\infty} (P_0 r_k^{-\alpha}) \cdot (2\pi r_k / x_0), \quad (14)$$

where $r_k = r_{k-1} + x_k$ with $r_0 = \gamma_f$ as a constant. $x_k = x$ is a random variable for $k \geq 1$. x and x_0 are both independent and identically distributed with a uniform distribution between r^l

and r^u , for $k \neq m$.

The proof can be found in Appendix II. This lemma shows that random positions of interferers outside a neighborhood follow similar characteristics to within the neighborhood.

Next, we derive the mean and the variance of n_{ij} 's. For analytical feasibility of obtaining the mean value, we approximate the joint probability density $P(r_1, \dots, r_{\infty})$ with a product form based on the Mean-field approximation [24].

Lemma 2: Consider $\{r_1, \dots, r_{\infty}\}$ as a random field. Based on the first-order Mean-field approximation, the joint probability density function $P(r_1, \dots, r_{\infty})$ can be approximated as a product of marginal distributions,

$$\prod_{i=1}^{\infty} P(r_i | \langle r_{i-1} \rangle), \quad (15)$$

where $\langle r_{i-1} \rangle = \gamma_f + (i-1)\bar{r}_c$ denotes the expected value of r_{i-1} . \bar{r}_c is an expected value of r_c , i.e., $\bar{r}_c = E(r_c) = \frac{r^l + r^u}{2}$, and $\langle r_0 \rangle = \gamma_f$.

The proof can be found in Appendix III. Using such a mean-field distribution, we obtain the expected value

$$E(n_{ij}) = \int_{r_1, \dots, r_{\infty}} \int_{x_0}^{\infty} \sum_{k=1}^{\infty} (P_0 r_k^{-\alpha}) \cdot \frac{2\pi r_k}{x_0} \cdot P(r_k | \langle r_{k-1} \rangle) P(x_0) dx_0 \dots dr_{\infty}, \quad (16)$$

where $P(r_1 | \langle r_0 \rangle) = P(r_1 | \gamma_f)$. Closed-form expressions can be derived in the following lemma for the individual terms in the sum.

Lemma 3: Let u_k be the term in the summation in (16), where $u_k = 2\pi P_0 r_k^{1-\alpha} = 2\pi P_0 (r_{k-1} + x_k)^{1-\alpha}$.

Denote the mean and the variance of u_k as $E(u_k)$ and $V(u_k)$ respectively. For $\alpha > 2$,

$$\begin{aligned} E(u_k) &= \frac{2\pi P_0 (\mathcal{D}_1^{2-\alpha} - \mathcal{D}_2^{2-\alpha})}{(2-\alpha)(r^u - r^l)}, \\ V(u_k) &= E(u_k^2) - E(u_k)^2, \end{aligned}$$

where $\mathcal{D}_1 = \gamma_f + (k-1)\bar{r}_c + r^u$, $\mathcal{D}_2 = \gamma_f + (k-1)\bar{r}_c + r^l$, and $E(u_k^2) = 4\pi^2 P_0^2 (\mathcal{D}_1^{3-2\alpha} - \mathcal{D}_2^{3-2\alpha}) / ((3-2\alpha)(r^u - r^l))$.

Let the k -th term of n_{ij} be $v_k = \frac{u_k}{x_0}$. Then, $E(v_k) = \frac{E(u_k) \ln(r^u/r^l)}{r^u - r^l}$, and $V(v_k) = \frac{V(u_k)}{r^u r^l}$.

The proof is given in Appendix IV. Using the results from Lemmas 1-3, we obtain the mean and the variance of the residual interferences below.

Theorem 1: For a link (i, j) , residual interference n_{ij} is a finite sum of independent random variables with different probability distributions. Thus, the probability distribution of n_{ij} is approximately normal with mean $E(n_{ij})$ and variance $V(n_{ij})$, where $E(n_{ij}) = \sum_{k=1}^{\infty} E(v_k)$, and $V(n_{ij}) = \sum_{k=1}^{\infty} V(v_k)$.

The proof is provided in Appendix V.

How accurate is the normal approximation? We check the modeling accuracy using Lyapunov condition. For n_{ij} to

converge to a normal distribution, the Lyapunov condition on $n_{ij} = \sum_{k=1}^{\infty} v_k$ should satisfy

$$\frac{(\sum_{k=1}^{\infty} E(|v_k - E(v_k)|^3))^{1/3}}{(\sum_{k=1}^{\infty} V(v_k))^{1/2}} \rightarrow 0. \quad (17)$$

Such condition is valid for an infinite sum of independent random variables. As the interference from far apart interferers is diminishing to zero, n_{ij} is in reality a sum of a finite number of independent random variables. As a result, the Lyapunov condition converges to a small constant instead of zero. A numerical analysis on the normal approximation is conducted on a network with infinite nodes, the increase of neighborhood size to k -th frontier neighbors, and channel attenuation parameter $\alpha = 4$. Figure 7 shows that the Lyapunov condition converges to a small constant around 0.97.

Hence, the advantage of the normal approximation is the simplicity. Only the mean and the variance need to be estimated. However, the approximation can deviate somewhat from a normal random variable measured by the Lyapunov condition.

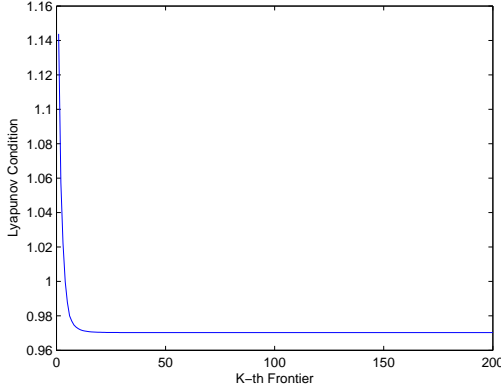


Fig. 7. Lyapunov Condition of the residual interference n_{ij}

C. Approximated Potential Energy and Probabilistic Model

The Mean-Field and the normal approximations result in approximated two network potential energy functions and thus the probabilistic models respectively. First, the Mean-Field approximation in Section V-A characterizes the residual interference as a deterministic quantity outside the neighborhood. The resulting system potential energy is

$$\begin{aligned} H^l(\sigma) &= \sum_{ij} g_{ij}(\sigma) \\ &= \sum_{ij} (a_{ij} + h_{ij}^*) \sigma_{ij} + \sum_{ij} \sum_{mn \in N_{ij}} a_{ij,mn} \sigma_{ij} \sigma_{mn}, \end{aligned} \quad (18)$$

where h_{ij}^* is deterministic and obtained from the mean-field equation. The corresponding Boltzmann distribution is

$$P^l(\sigma) = Z_{\sigma}^{-1} \exp(-H^l(\sigma)). \quad (19)$$

Such a Boltzmann distribution is a second-order Markov Random Field with deterministic coefficients a_{ij} 's, h_{ij}^* 's and

$a_{ij,mn}$'s. The resulting probabilistic dependency graph is shown in Figure 8.

Second, the normal approximation characterizes the residual interference outside the neighborhood as a Gaussian random variable that aggregates contributions from (unseen) interferers. The resulting system potential energy is,

$$\begin{aligned} H^l(\sigma, \mathbf{n}) &= \sum_{ij} (-R_0 + \frac{N_b}{P_i l_{ij}^{-\alpha}}) \sigma_{ij} \\ &+ \sum_{ij} \sum_{mn \in N_{ij}} \frac{P_m l_{mj}^{-\alpha}}{P_i l_{ij}^{-\alpha}} \sigma_{ij} \sigma_{mn} + \sum_{ij} \frac{n_{ij}}{P_i l_{ij}^{-\alpha}} \sigma_{ij} \\ &+ \beta \sum_{ij} U(R_{ij}^l - R_{th}) \sigma_{ij}, \end{aligned} \quad (20)$$

where \mathbf{n} is a random vector including all n_{ij} 's. The corresponding Boltzmann distribution is

$$P^l(\sigma, \mathbf{n}) = Z_{\sigma}^{-1} \exp(-H^l(\sigma, \mathbf{n})). \quad (21)$$

This approximated model is a random-bond Markov Random Field [16], where n_{ij} 's are random coefficients (bonds) for σ_{ij} 's.

As distributed decisions only need to determine the values of σ_{ij} 's, the marginal probability model for σ_{ij} 's is

$$P^l(\sigma) = \int P^l(\sigma, \mathbf{n}) d\mathbf{n}. \quad (22)$$

VI. MESSAGE-PASSING OVER FACTOR GRAPH

A simplified probabilistic model $P^l(\sigma)$ from either the mean-field or the normal approximation can be used to determine link activities,

$$\hat{\sigma} = \arg \max_{\sigma} P^l(\sigma). \quad (23)$$

Randomized and distributed decisions implement such optimization through message passing over localized factor graphs. The general message passing algorithm is provided in [23] and the references therein. The novelty here is to couple message passing with approximation of the residual interference. The corresponding functional nodes and variable nodes are chosen accordingly to couple the approximations with distributed scheduling.

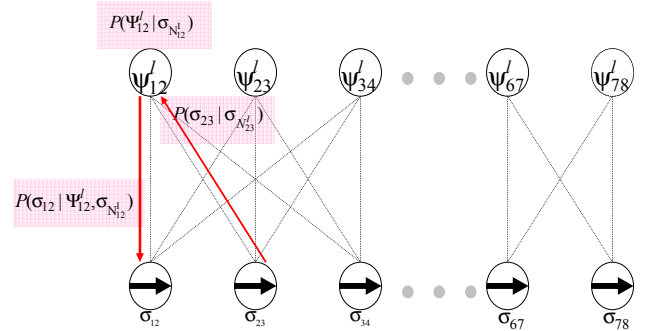


Fig. 8. Example Factor Graph with Functional Nodes and Variable Nodes.

A. Message Passing with Normal Residual Interference

The local potential function in message passing, $P(\psi_{ij}^l | \sigma_{N_{ij}^I})$ for link (i, j) , exhibits a special form from approximating residual interference as a normal random variable, i.e.,

$$P(\psi_{ij}^l | \sigma_{N_{ij}^I}) = N(E(\psi_{ij}^l), V(\psi_{ij}^l)), \quad (24)$$

where $\psi_{ij}^l = R_{ij}^l$. $N(E(\psi_{ij}^l), V(\psi_{ij}^l))$ denotes a normal distribution with a mean

$$E(\psi_{ij}^l) = \frac{\sum_{|X_m - X_j| \leq \gamma_f} P_0 l_{mj}^{-\alpha} \sigma_{mn} + N_b}{P_0 l_{ij}^{-\alpha}} + \frac{1}{P_0 l_{ij}^{-\alpha}} E(n_{ij}), \quad (25)$$

and a variance (from (2))

$$V(\psi_{ij}^l) = \left(\frac{1}{P_0 l_{ij}^{-\alpha}} \right)^2 V(n_{ij}). \quad (26)$$

Message from a function node ψ_{ij}^l to a variable node σ_{ij} is

$$P(\sigma_{ij} = 1 | \psi_{ij}^l, \sigma_{N_{ij}^I}) = U(R_{th} - \psi_{ij}^l), \quad (27)$$

where $U(R_{th} - R_{ij}^l) = 1$, if $R_{th} > R_{ij}^l$; 0, otherwise.

Message from a variable node σ_{ij} to a function node ψ_{mn}^l is

$$P(\sigma_{ij} = 1 | \sigma_{N_{ij}^I}) = \begin{cases} \frac{1}{2} + Q\left(\frac{R_{th} - E(\psi_{ij}^l)}{V(\psi_{ij}^l)}\right), & E(\psi_{ij}^l) \leq R_{th} \\ Q\left(\frac{E(\psi_{ij}^l) - R_{th}}{V(\psi_{ij}^l)}\right), & \text{otherwise,} \end{cases} \quad (28)$$

where $Q(x) = \int_0^x \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$. This equation is obtained from an equality of $P(\sigma_{ij} = 1 | \sigma_{N_{ij}^I}) = \int P(\sigma_{ij} = 1 | \psi_{ij}^l, \sigma_{N_{ij}^I}) P(\psi_{ij}^l | \sigma_{N_{ij}^I}) d\psi_{ij}^l$.

Consider an example in Figure 8. The second node at the top row determines the scheduling of link (2,3). This node maintains only the local factor graph related to link (2,3). The local graph has variables ψ_{23}^l , σ_{12}, σ_{23} and σ_{34} .

The algorithm includes two steps: (a) estimating the parameters for the mean and variance of the residual interference, (b) using the estimated parameters in the local functions for message passing. For example, in Figure 8, a functional node Ψ_{12}^l estimates the mean and variance of residual interference n_{12} , $E(n_{12})$ and $V(n_{12})$, from the received message from neighboring variable nodes σ_{23} and σ_{34} . The estimation is from Theorem 1.

The functional node ψ_{12}^l sends to a variable node σ_{12} a message $P(\sigma_{12} = 1 | \psi_{12}^l, \sigma_{N_{12}^I}) = U(R_{th} - \psi_{12}^l)$. The variable σ_{12} sends to all the neighboring functional nodes, Ψ_{12}^l , Ψ_{23}^l and Ψ_{34}^l , a message $P(\sigma_{12} = 1 | \sigma_{N_{12}^I})$ in (28).

B. Message Passing with Mean-Field Residual Interference

The local function with the Mean-Field approximation of the residual interference, i.e., h_{ij}^* in (8), is relatively simple,

$$P(\psi_{ij}^l = a_0 | \sigma_{N_{ij}^I}, h_{ij}^*) = \delta(\psi_{ij}^l - a_0), \quad (29)$$

where $\psi_{ij}^l = R_{ij}^l$ in (2) with $\text{Res}_{ij} = h_{ij}^*$ in (8). a_0 is a constant. $\delta(\psi_{ij}^l - a_0) = 1$, if $\psi_{ij}^l = a_0$; 0, otherwise.

Message from a function node ψ_{ij}^l to a variable node σ_{ij} is

$$P(\sigma_{ij} = 1 | \psi_{ij}^l, \sigma_{N_{ij}^I}, h_{ij}^*) = U(R_{th} - \psi_{ij}^l), \quad (30)$$

where $U(x)$ is a unit step function.

Message from a variable node σ_{ij} to a function node ψ_{mn}^l is

$$P(\sigma_{ij} = 1 | \sigma_{N_{ij}^I}, h_{ij}^*) = U(R_{th} - \psi_{ij}^l). \quad (31)$$

The overall algorithm includes two steps: (a) estimating the mean-field parameter h_{ij}^* 's, and (b) obtaining decisions σ_{ij} 's through message passing. These two steps alternate until convergence.

VII. SIMULATION

We now evaluate network-wide link outage probability to assess the benefit of the approximation on link-scheduling. As the link-outage probability is not analytically related yet with the network model, we conduct simulations for the shared channel access on a multi-hop wireless network.

A. Simulation Setup

Network nodes are positioned uniformly in a square area of 100 square meters, and composed of $L = 200$ links. We consider $2 \leq \alpha \leq 6$ for the channel attenuation factor. Here, $2 \leq \alpha \leq 3$ results in 'strong' dependence among interferers. This corresponds to the scenario which is unsolved in our prior work. We choose $10 \leq \text{SINR}_{th} \leq 100$. We consider the maximum spatial channel-reuse at a time instance.

B. Simulation Results

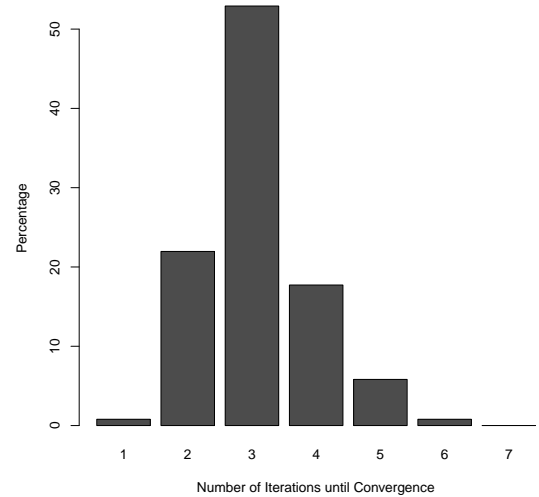


Fig. 9. Convergence of Distributed Link-Scheduling Decisions

Using the above experimental setting, we first study through simulation whether distributed decisions converge sufficiently fast. Figure 9 shows that the distributed link-scheduling decisions converge through interactions among neighbors in a few iterations.

We now compare the performance of the following algorithms: (a) distributed link scheduling with the Mean-Field approximation, (b) distributed link scheduling with the normal approximation, and (c) the conventional distributed link-scheduling algorithm [2][9][16] that simply ignores the residual interference outside neighborhood range. The link outage probability is used as a performance measure. Such a performance measure assesses the probability that a link cannot satisfy the SINR requirement. Here, the link outage is mainly caused by the limited neighborhood size. There, improved performance signifies the importance of proper approximations of information outside a neighborhood.

Figure 10 plots the outage probability as a function of neighborhood size γ_f assuming that all nodes have the same size neighborhood. This figure shows that our two approximations for the information outside neighborhoods result in a small link outage probability given for a small neighborhood range γ_f . The conventional scheme, however, has a large link-outage probability for a small γ_f . This shows that for a small neighborhood, learning the residual interference with a loss variable provides a big gain. The performance with the mean-field approximation varies with respect to the measurement error of residual interference. When the measurement error is small, the Mean-Field approximation results in a small link-outage probability across different neighborhood sizes. Overall, the two approximation approaches outperform significantly the method that ignores the residual interference, showing the importance of approximating the information outside a neighborhood.

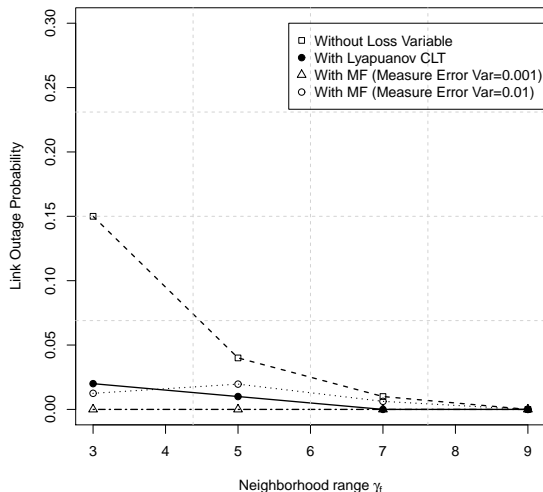


Fig. 10. Link Outage probability with $\text{SINR}_{th}=10$. Dotted line: Conventional scheme. Other curves: Our approximation approaches

Figure 11 shows how the link-outage probability varies with respect to channel attenuation factor α . The link outage probability for the two approximation schemes remains small for different α . As α increases, the residual interference outside the neighborhood decrease, and the link outage probability of the two approximation schemes approaches zero.

It is worth mentioning that for small α , e.g., $\alpha = 3$, the two approximation approaches significantly outperform the conventional approach. The scenario of small α is shown in our prior work [16] to be difficult for distributed scheduling with information only from neighbors. Hence, the result here shows the importance and the ability of the approximations to account for the aggregated residual interference outside a neighborhood.

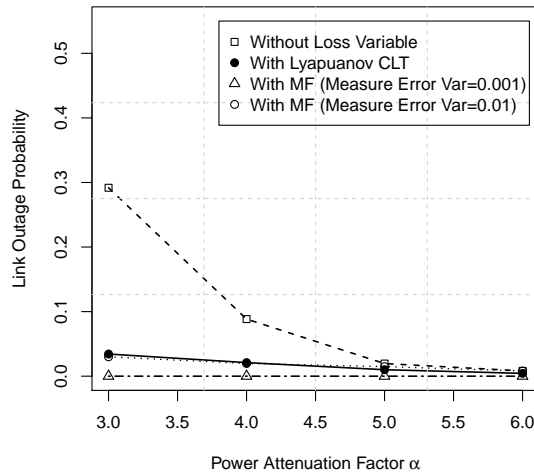


Fig. 11. Link Outage probability with $\gamma_f=4$ and $\text{SINR}_{th}=10$. Dotted line: The conventional scheme. Other curves: The two approximation schemes.

Figure 12 and 13 show the performance as a function of SINR. As SINR threshold increases, the link outage probability increases proportionally except for the Mean-Field approximation with small measurement error. The performance of the Mean-Field approximation for a large measurement error is worse as the SINR threshold increases. This is because the SINR constraint is more easily violated due to the measurement error as the SINR threshold increases.

For the approximation scheme with Lyapunov central limit theorem (CLT), once γ_f is moderately large, the proposed scheme copes well with a large SINR threshold. When neighborhood size γ_f is not sufficiently large, for a moderate SINR threshold, conventional schemes in the literature often fail to satisfy the SINR requirement, resulting in a high link outage probability. On the contrary, across a wide range of SINR threshold values, the proposed scheme with Lyapunov CLT performs efficiently and improves the link outage probability significantly. In particular, if γ_f is moderate (e.g., $\gamma_f = 4$ in Figure 13), the link outage probability quickly approaches zero. Therefore, with both approximation approaches, resource utilization is significantly improved overall.

VIII. CONCLUSION

We have studied in this work distributed scheduling decisions where each wireless node uses complete information within a neighborhood and inferred information outside. We

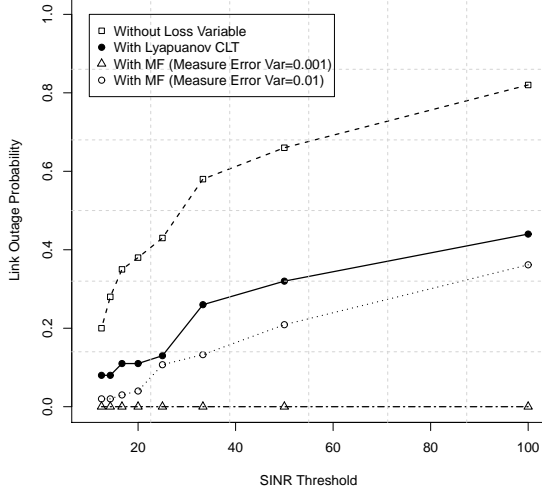


Fig. 12. Link Outage probability with $\gamma_f=3$. Dotted line: Conventional scheme. Other curves: Approximation schemes.

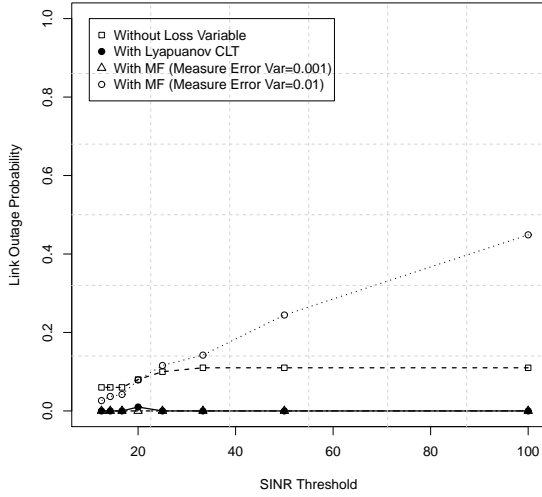


Fig. 13. Link Outage probability with $\gamma_f=4$. Dotted line: Conventional scheme. Other curves: Approximation schemes.

have characterized the complex spatial dependence of distributed link-scheduling decisions with a factor graph. The factor graph enables information exchange among neighbors. More importantly, the long-range dependence in residual interference is not ignored but approximated by one variable as the aggregated dependence outside a neighborhood. The Mean Field approximation considers the residual interference outside a neighborhood as a deterministic Mean-Field. Such Mean-Field approximation optimizes the variational free energy, resulting in a simple Markov Random Field model for distributed decisions. A disadvantage is that the Mean-Field estimates can be computationally costly to obtain. The normal approximation considers the residual interference outside a neighborhood as a random loss variable. Such a loss variable is

approximately normal by the Lyapunov central limit theorem, resulting in a simple Random Bond model for distributed scheduling decisions. A disadvantage is that the condition for the asymptotic normality can be somewhat deviated for a network with a finite size.

The approximated dependencies are easily incorporated into the factor graph. This results in an extended message-passing algorithm with the two phases. First, the approximations are obtained iteratively from the available information at neighboring nodes. Second, the approximations are combined with messages from neighboring nodes and allow each node to make its link-scheduling decisions. The algorithm alternates between the two phases, and has been shown to converge in a few than 10 iterations.

We have shown through simulations that the link outage probability of the resulting link-scheduling decisions is significantly improved by the approximated information. The improvement is especially pronounced for distributed decisions with a small number of neighbors and a slowly-varying channel, where the link-outage probability is reduced from, e.g., 0.3 to 0.04.

An open issue as possible future work is to derive an analytical relationship between the network-wide link outage probability and the approximated information.

APPENDIX

I. DERIVATION OF EQUATION (13)

$\mu_{ij} = P(\sigma_{ij} = 1)$, where the probability is evaluated using the simplified Boltzmann distribution $Q(\sigma)$ from the mean-field approximation. Expanding the expression of μ_{ij} , we have

$$\mu_{ij} = \frac{\exp(-h_{ij}^*)\eta_1}{\exp(-h_{ij}^*)\eta_1 + \eta_2}, \quad (32)$$

where $\eta_1 = \exp(-a_{ij}) \sum_{(m,n) \neq (i,j)} \exp(-H(\sigma \setminus \sigma_{ij}))$ with $\sigma \setminus \sigma_{ij}$ being all σ_{mn} 's except for $(m,n) = (i,j)$. $\eta_2 = \sum_{(m,n) \neq (i,j)} \exp(-H(\sigma | \sigma_{ij} = 0))$.

Representing h_{ij}^* from the above expression, we have

$$h_{ij}^* = \log \frac{1 - \mu_{ij}}{\mu_{ij}} \frac{\eta_1}{\eta_2}. \quad (33)$$

Replacing this expression in (13), we have

$$\log \frac{1 - \mu_{ij}}{\mu_{ij}} \frac{\eta_1}{\eta_2} = \sum_{mn \notin N_{ij}} a_{ij,mn} \mu_{mn}. \quad (34)$$

Note that η_1 and η_2 are functions of h_{mn}^* 's (μ_{ij} s) for $(m,n) \neq (i,j)$. μ_{ij} is also a function of h_{mn}^* 's. Hence, the equation is coupled. In addition, there are multiple equations that contain common h_{ij}^* 's. Hence, for all (i,j) 's, the mean-field equations are coupled, and can be solved iteratively.

II. PROOF OF LEMMA 1

On the k -th circle for $k \geq 1$, the total number of active links on the circle is $\frac{2\pi r_k}{x_0}$, which is random. Each active link on the k -th circle results in as much interference as $P_0 r_k^{-\alpha}$. As a result, total interference from the active links on the k -th circle is $(P_0 r_k^{-\alpha}) \cdot \frac{2\pi r_k}{x_0}$; and, thus $n_{ij} = \sum_{k=1}^{\infty} (P_0 r_k^{-\alpha}) \cdot \frac{2\pi r_k}{x_0}$.

III. PROOF OF LEMMA 2

Mean field approximation shows that the joint distribution of a random field (y_1, \dots, y_n) can be approximated by a product of marginal distributions. Thus, $P(y_1, \dots, y_n) = \prod_{i=1}^n P(y_i | \langle y_{N_i} \rangle)$ with a good approximation, where y_{N_i} is the set of neighboring random variables of y_i . From the assumptions, $r_k = r_{k-1} + x_k$. Thus, $P(r_k | \langle r_{N_k} \rangle) = P(r_k | \langle r_{k-1} \rangle)$.

IV. PROOF OF LEMMA 3

The average of u_k , denoted with $E(u_k)$, can be derived from an integration of $\int 2\pi P_0 r_k^{1-\alpha} P(r_k | \langle r_{k-1} \rangle) dr_k$. The average of u_k^2 can be derived in a similar way.

V. PROOF OF THEOREM 1

Consider the configuration in Figure 6. For an active link (i, j) , the residual interference of this configuration is denoted with a random variable n_{ij} , which is a function of random variables r_k for $k \geq 1$, where r_k is the radius of the k -th frontier.

Note that r_k 's are independent random variables with different uniform distributions. Lyapunov's central limit theorem [32] shows that the summation of a large number of independent random variables (even with different distribution) results in a normal distribution. n_{ij} is a summation of hundreds and thousands of independent random variables (i.e., r_k 's), thus n_{ij} can be approximated by a normal distribution.

Furthermore, a normal distribution can be completely characterized by mean and variance. The mean and variance of n_{ij} is denoted with $E(n_{ij})$ and $V(n_{ij})$, respectively, i.e., $E(n_{ij}) = \sum_{k=1}^{\infty} E(v_k)$, and $V(n_{ij}) = \sum_{k=1}^{\infty} V(v_k)$.

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